Journal of Current Medical Research and Opinion

CMRO 09 (04) 4739-4756 (2026)
DOI: https://doi.org/10.52845/CMRO/2026/9-4-1
www.cmro.in

CMRO
ISSN (O) 2589-8779
(P) 2589-8760

Original Research

Mathematical Foundations of Artificial Intelligence in Criminal Justice and how they are applied in Forensic Analysis

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Abstract

In this review, we have considered the mathematical concepts that may be of most usefulness in the view of crime science. It will start by giving a high-level overview of the methodologies that are likely to be used with special attention given to complexity science methods. One will find out how mathematics is stimulated in image processing, and how image processing can improve the methods in mathematics. Mathematics is the keystone of any artificial neural network, and the initial steps to its construction can be made through the study of the principles of neural networks. Forensic analysts use mathematics to rebuild crimes, evaluate evidence, and compute timing so as to offer objective, quantitative, and scientifically valid methods. It uses trigonometry, probability, and statistics in areas of toxicology, DNA profiling, and bloodstain analysis to transform physical clues to evidence-based conclusions. This chapter addresses the issue of forgetting the prerequisites by the researchers. You will have a birds-eye view of the matter in this chapter as you find out the meaning of some of the requirements such as Image processing mathematics, forensic image processing mathematics (which comprises the basics of the neural networks) and prob ability theory. The forensic sciences extensively make use of the concepts of probability density. The concepts that were discussed during the briefing seem to belong to different areas of research but, in fact, are highly interrelated and will provide the reader with a general idea about the given topic. Section two provision of links between mathematics, image processing and forensic science and Section one is dedicated to mathematics of image processing.

Keywords: Mathematical Foundations, Forensic Analysis, Criminal Justice, Artificial Intelligence.

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Received: 02.01.2026 | **Revised:** 27.03.2026 | **Accepted:** 08.04.2026 | **Published:** 14.04.2026

Introduction

It will make sense to begin with determining the exact meaning of the word mathematical because that is how the term has been applied in this review in a general way. Even a brief definition, like that of the study of quantity or the study of patterns, is so broad as to make no sense since the term applies to such a diverse range of methods and approaches. The question in this case is how to find out which among the numerous variants of thought and application of mathematics can really have a difference in the sphere of the criminal justice. Statistical analysis is an inseparable part of almost all quantitative studies, e.g. [1, 2], and this is not the only example of how mathematics is already present and extensively used in the sphere of criminal justice. Whereas statistics does have a role to play on such work, it is predominantly utilitarian, which it works to support the other forms of analysis. Using statistical methods in crime data analysis is no different than applying such methods to other areas; however, the use of the statistical methods is seldom a priority in criminological studies. Though the applied methods are undoubtedly sophisticated, the approach is not based on a completely mathematical position. In case mathematical knowledge can manage to encode the functional relationships, it will be able to make a distinctive contribution. The term functional plays a crucial role here as the aim of the methods is to describe how quantities are correlated and not to demonstrate how quantities are numerically related to each other. As an example, rather than stating that the longer the sentences are, the lower the crime rate, such a presentation could demonstrate the impact that a sentencing has on the cost-and-benefit analysis of individual criminals and indicate the overall result. The duality of real-life processes and their models allows analyzing the real functioning of the system and taking into consideration all the instrumental relationships. In a nutshell, this process is referred to as mathematical modelling. Towards the end of his career in 1940 the Cambridge mathematician GH Hardy wrote A Mathematician;s Apology, which is generally seen as a captivating account of the teaching of mathematics [3-5]. One of the underlying concepts of the article is the highest level of respect, which is extended to pure mathematics with no practical problems. Others have even termed it as a defense of mathematics as art-for-art-sake in the sense that there is no really great mathematics which has any practical application. Although he denies it, Hardy does tell us that nothing that I ever did has the least practical use, which collides with some of the main principles of criminology. The fact that this topic has been included in this handbook might be astonishing, hence, in case this is indeed the attitudes and contributions of mathematicians [6, 7]. Conversely, the ideas presented by Hardy are so outdated that the applied research is the rule and not an exception in the scientific community of the present days. Intriguingly, the distinction between the two forms has been demonstrated to be misplaced, particularly in the subject of number theory pioneered by Hardy the subject of his study, namely, the intrinsic property and relations of numbers.

The encryption systems which secure all transactions performed online are founded in what at first was thought the least applicable area; this, in many regards, is the most significant security that is provided in the modern world. The use of mathematics in technology and the study of physical systems is rather evident; while its use on social ills such as crime is not as evident. It has always been believed that social systems cannot be mathematically analyzed because they are quite complex. The acts and interactions of the human beings are more complex than the rules that govern the physical systems, and are the center of the phenomena in question. However, a number of ways and approaches have developed over the past few years to address the challenges introduced by such complexity. These form the basis of future mathematical developments and have created a new enthusiasm to apply mathematics in solving previously unsolvable problems. One of such topics has attracted much interest in the modeling and analysis of crime within the mathematics community in recent times. Mathematics can be applied in a number of fields of crime science. It provides a means of describing concepts and associations in real-world terms since the discipline itself is preoccupied with formalization and rigor [8]. At the simplest level that may simply ease other tasks, including providing a framework of learning social networks. However, where the technique is really effective is in modeling, or applying behavioral mechanisms in reduced mathematical form. The situations they describe, as well as what possible happens and the impact of changes, can be more comprehended by examining the nature and conduct of such models. The importance of mathematical treatment would be much more evident when we remember that much of the critical concepts in the crime science such as as intervention, assessment, and prediction can be formulated in such terms. The approach is complementary to the ability of the other fields that also contribute to it because it introduces a differentiating viewpoint that is logical and rigorous, an integral part [9].

Techniques of enhancing AI can be classified into three levels, which are incremental design, specific design application, and step change. To understand and implement AI, the presence of satisfying mathematical and statistical notions is extremely sought after. In this case, we attempt to illustrate the relationship in a simplistic fashion with a small number of themes. Only some explanations are provided with detailed references on the future research as they are beyond the frames of the chapter. Since mathematics, image processing and forensic sciences are all linked to each other, this course would equip the student with the background knowledge on which to start exploring the potential of forensic image processing. Multimedia forensics uses artificial intelligence where convolutional neural networks are used. Through machine learning, forensic image identification can dissect fake data on photographs; forensic anthropology is also used to a great effect in medicine. The concepts of picture sampling, augmentation, convolution, and neural networks are also contained in this chapter to enable the reader to understand these contemporary studies in multimedia. The mentioned statistical methods are also utilized in forensic investigation that is based on the Internet of Things. The methodical development is traced using score-based likelihood that is applied in forensic studies, beginning with extremely basic mathematical forms and image processing. This article could be of interest to anyone willing to learn more about the quantitative method of weighing evidence based on likelihood ratio [10, 11]. Towards the end of the chapter, an overview of the neural networks is given, which forms the basis of digital forensic neural networks. With the above consideration, the first section gives a brief discussion of different issues associated with mathematical representation and image processing, including the image formation model and image sampling, intensity transformation and spatial filtering, image enhancement using Laplacian mask, image denoising, order statistics filters, and convolution in image processing. The second section is on mathematical and forensic image processing. Among subjects, there are steganography, score-based likelihood, discrete fourier transform and an overview of neural networks and probability theory in general. Geometric and statistical systems, not to mention probabilistic modeling, have been instrumental in the analysis of physical patterns and impressions. A false conviction of Ray Krone due to incorrect evidence of bitemarks resulted in the realization of the issues with subjective analysis of pattern and the need to find objective indicators of reliability. With affine transformations and morphometric modeling by considering geometric distortion, comparisons of dental impressions are more accurate and fewer false positives are obtained. Due to the same reason, trigonometric models enable an analyst to establish the angle of impact of blood droplets based on the geometry of the stain in the analysis of bloodstain patterns. These methods have enhanced the scientific basis of pattern analysis and emphasize the need to continue with the validation and training [12].

It is also in the case of forensics that errors in the application or interpretation of mathematical reasoning have caused severe outcomes. The fallacy by the prosecutor of confusing the probability of seeing the evidence with innocence with the probability of innocence with the evidence and the misuse of conditional probability is both indicated in the People v. Collins case. A similar statistical error which contributed to the wrongful conviction of Sally Clark was the fact that a very wrongly small joint probability estimate was obtained because of the assumption that two unexpected child deaths were independent. Technical knowledge that can be used to teach probability understandings to normal people is required, and legal professionals should be statistically literate as these cases demonstrate. Recent advancements in AI and ML have resulted in new possibilities and problems in the field of forensic science. The face recognizer, handwriting recognition, deep fakes, and authorship verification are not the only contemporary uses of such methods as Natural Language Processing (NLP) and Convolutional Neural Networks (CNNs). In order to confirm or deny alibi, in the Karen Read case, digital timeline reconstruction was applied to time-series analysis and device metadata. Likewise, ML models have also been developed to identify manipulated media based on the detection of statistical anomaly over video frames. Such models employ Principal Component Analysis (PCA) and Singular Value Decomposition (SVD). However, when put into contested legal contexts, the inability to see the inner workings of some AI systems casts doubt on the concept of reproducibility, interpretability, and algorithmic bias.

Mathematics is also important in postmortem investigation. It is estimated that Postmortem Intervals (PMI) can be calculated using the rate of body temperature fall as a result of solving a differential equation, namely the thermal decay prediction equation (e.g., the Newton Law of Cooling). These experiments run by Jill Dando and Harold Shipman did reveal however that other factors besides the control of the individual may cause significant changes on the rate of decomposition like humidity levels, body mass and body temperature. With that in mind, it is evident that the use of empirical data that is gathered in real-life crime scenes and multidisciplinary teams is indispensable to the continuous enhancement of models [12]. Forensic identification has adopted a greater role in providing more uses of mathematics because genetic genealogy has emerged as a method of investigation. The Golden State Killer case is one of the high-profile cases that used the kinship coefficients and pedigree likelihood model to match the DNA found at the crime scene with long-lost family members in the genealogy records. Management of ruined DNA and poor family trees on the mass graves in Bosnia were particularly difficult to analyze, although Bayesian networks and combinatorics were useful in identifying the victims. There is a great demand to have computing systems that can be able to properly manage complex and multi generational relationship information and these methods represent that. Sequential tests, control charts, and anomaly detection algorithms have been another provision of the healthcare facilities to detect unusual trends of death. The Harold Shipman case demonstrates that CUSUM (Cumulative Sum Control Charts) may be used to track the death rates in a statistical manner and detect the abnormal trends before they can be identified using more traditional qualitative approaches. Toxicological timelines are built by relying on pharmacokinetic equations which describe the ADME processes in order to achieve the likely time and cause of death in cases of overdose. There is also a basic problem with these gains despite these gains in the statistical literacy of the judges, attorneys and jurors. Complex mathematical results should be subject to expert interpretation since they may be easily misunderstood or misinterpreted. As a result, to avoid injustice, and ensure the credibility of the court decisions, it is increasingly becoming important to have the forensic statisticians at hand and to have the experts testify in a more structured and transparent way [13]. This is the age of more sophisticated adversarial approaches and data-rich digital evidence where the need is to have verifiable mathematical models that can undergo scrutiny and be tested. The possible future actions involve developing certification of forensic algorithms, developing explainable AI models, and educating more mathematics in forensic educational courses. To sum up, mathematics is not a helpful companion in evidence analysis in the field of forensic science but rather a significant component thereof. When applied appropriately, it makes the judicial process more enriched by enhancing objectivity, reducing the bias, and adding transparency. The application of mathematical methods is indispensible in forensics studies as the cases and approaches that have been surveyed prove.

Besides enhancing the impartiality, quantitative models enhance the reliability and admissibility of forensic evidence in a court of law. An illustration is the way Bayesian statistics has modified the methods of communicating uncertainty in the interpretation of evidence. Rather than the yes/no answers, it provides the likelihood ratios. Probability and statistical rigor have been used to improve transparency especially in the area of trace analysis and DNA profiling. Biometric recognition and document verification, predictive policing and crime linkage are all the fields of forensic workflow that are becoming more and more dependent on machine learning and artificial intelligence technologies. However, to prevent biases of algorithms and ensure their possibility to be brought to life, such technologies must be rigorously tested and ethically controlled. This is also true of postmortem interval estimation with differential equation model: they must be continually optimized, tested by other fields, and take into consideration evolving environmental conditions. The other concern that is also increasing is that jurors and legal professionals may not be statistically literate and this may make them misinterpret probabilistic outcomes. Hence, it requires forensic statisticians and more understandable expert testimony. To keep up with the latest technology, more complex information, and evolving methods of adversary, forensic mathematics must also evolve to make it robust and reliable in court [13, 14]. This literature review will attempt to explore the mathematical basis of AI, how it has been applied in solving crimes, and its role in the areas of forensic science and criminal justice.

Mathematical Methods and Violent Crime

The assumption that the problem of crime cannot be modeled mathematically is not a new concept. This point of view is largely due to the fact that a great number of tricky behaviors are involved; it is not that simple to explain crime with a few equations. But research dealing with just such large-scale systems of the real world has been spread like wildfire over the past few years. These specialized methods do assist us in making some progress, although such will never be so readily analyzed as more orthodox subjects. The term complex system is generally defined by describing an example or by listing its broad characteristics because of the broadness of the discipline. All operational definitions have three common characteristics but the degree of emphasis on each of them changes depending on the situation under consideration. The aggregate behavior of the system is not directly proportional to the mechanisms of the system at the micro level. It is the third of these characteristics, the so-called emergent behavior, that actually makes complex systems different. These phenomena could include the emergence of patterns up to the totally chaotic behavior, and they are practically always non-linear. All these have the common attribute of being irreducible which implies that they can be observed only on a high level. In my opinion it is rather obvious that this is one of the legitimate perspectives of criminal phenomena. In the classic broken windows theory, which gives a very tempting example of this principle in practice, it is said that endemic and possibly more heinous crime is carried out at the area level because of an individual-level effect, the change in perception of the offender into a change brought about by the environmental mental stimuli. The non-linearities and feedback loops that are embedded in complex social systems are defined, in the context of crime in general, by the fact that there are unseen consequences and exercising control becomes challenging. Some of the diverse areas where complex systems are observed include ecology, economics and urban studies. However, since we are dealing with criminals in this case, it is the most reasonable to outline the most common methods. It has mathematical origins, but the field is inherently multidisciplinary and employs techniques of computer science, economics, and physics [14]. This is where we will list some of them that are of specific concern in the criminal justice.

Evolving Structures

One of the major areas of the dynamical systems theory is mathematical explanation of how quantities or things vary over time. Certainly there are numerous parallels between this field of complexity research and the classical applied mathematics, and the models that are employed in it resemble those that are employed in other fields, including fluid mechanics. The overall objective is to demonstrate the evolution of selected characteristics of the system, which is depicted by variables that have specific definitions (such as variables such as f and g, etc.). In the majority of cases, the variables that are described by the help of the differential equations are considered as continuous-valued, which implies that these variables may assume a continuous set of values. The rate of change of a variable with respect to itself and other variables is determined by these equations which are the hypothesised mechanism. Typically, an equation will be formulated about each variable of interest and it is these rates of change that determine how the behavior will behave over time. These taken together give an equational representation in which the model is stored. There are a couple of equations of varying types where the change of one variable is a rate of another; the interaction between the two variables is referred to as coupling [15]. These connections result in interesting behaviours and the specific behaviours of complex systems are largely based on the complex pattern of coupling. But, due to the fact to the mature region and abundance of advanced methods, equation-based models are some of the most amenable in regard to mathematics. By mathematically analysing dynamical systems, you can learn a lot about the stability of a system and patterns it forms and the way it evolves with time. A strong solution is said to be stable with unstable solutions that are likely to undergo a substantial qualitative change; stability can be referred to as the effect of minute perturbations to the system. There are serious practical implications of these problems: whatever form of instability is a bad sign of any system and may have disastrous consequences. This type of modeling is best used to study criminal problems which are dynamic in nature including aggregate criminal rates, incarceration levels and geographical distributions. A not-criminal amount which can be factored in a model of metal theft would be commodities prices. The dynamic variables may be the national crime rate (macro-level) or micro-level. The hard thing is to derive equations of hypothetical behaviors, but once one has them, one can study behavior (or predict) at a finer level.

Computer Systems

One of the areas that have been strongly associated with complexity science is the study of networks. A network is simply a set of individual nodes (also known as vertices) and the connections (also known as links) which exist between these nodes. Many of the terms are borrowed out of "graph theory" which is a sub-discipline of mathematics and networks are modeled after real-life graphs. A good example will be social networks which are some of the most popular types of networks. These networks are depicted by actors, or individuals, and the social interactions, or connection, among actors are denoted by such things as friendship, communication, and physical touch. Examples of these numerous types of networks, include telecommunications (such as the internet), transportation (such as air travel), and ecology (such as food webs). Network representations are able to express a great number of relationships due to their malleability. Indirectly, directed networks can be directed and weighted networks can be directed and linked with a value (contact frequency) [16]. Network studies involve analyzing real life networks, and adapting them to theory. The former type of research majors on the definition of network topologies, i.e. by computing the centrality of individual nodes. The number of linkages that a node possesses is among numerous straightforward ideas of centrality; such ideas lay diverse stress on different sections of the framework. There are many examples of studies of real-world networks and they all have the common statistical characteristics. This type of analysis can be used to ascertain the significance of a node or the role that it is playing in the network. The related area is community detection, which is supposed to locate important groups of nodes. Criminal situations that involve networks are numerous. A number of sub areas in criminology revolve around the analysis of social relationships; an example of this is that whilst organized crime, by far, is the most common, one can also use network based approaches to study the relationship between victims and offenders. Besides providing a convenient structure of data, analysis may help illuminate the activities of various actors (e.g., brokerage or command) and reveal organizational fingerprints. There are other criminal issues that are related to networks. The built environment may significantly influence the occurrence of geographical events because transport networks are one of the main determinants of the built environment. The issues of networks are very critical in the issue of resiliency of the infrastructure (e.g., electricity networks) and are to be considered when the risk is being profiled, in a wider security context.

Domains of Implementation

Spatio-Temporal Patterns

Mathematical analysis of crime patterns in space and time is one field where empirical findings in geographical criminology have been stimulated. The fact that it is a stylized process that may be compared to another physical process makes it an ideal object to modeling due to the identification of dynamic patterns in crime data, like hotspots and near-repeat victimization. The apparent spreading of criminal activity is so tempting to be paralleled to the corresponding formulations of the pattern development, as well as the simple physical diffusion processes. Various hypotheses of behavior can be used to explain the patterns of crime in urban areas, some of them having an environmental factor. Thus the modeling method is straightforward: write down these hypotheses mathematically and calculate whether they can generate patterns the same way as the empirical ones. This may prove helpful in numerous aspects in case a practical model could be located. In simple terms, it reveals that the patterns of behavior that are explained by the hypotheses are, in fact, observed in the real world. Further, through the examination of such a model, we are able to acquire additional insight into the situations which lead to patterns and the manner in which they evolve. The latter is likely to be the most critical of them because it opens a perspective of extrapolating patterns into the future in the case when their dynamics can be comprehended. The possible usefulness of this to preventive effort is apparent, as it amounts to geographical prediction.

The simplest form of such a model is the one, which is founded on the behavior of individual actors: burglars move between grid-positioned residences, where they are attracted by the ones, which are more attractive with their ratings. In case physical attractiveness is not sufficient, they engage in crimes that heighten the value of the property that was victimized temporarily. Notably, the authors can then describe this model which is, in essence, an agent-based one with the help of differential equations. The circumstances of the formation of hotspots and their direct shape and development may be ascertained by mathematical calculations. This is better than the findings made by agent-based simulations where one can only experience such events qualitatively. One of the extensions and changes to the Short model is incorporation of police action. The proposed network-based model can be compared to the rest, which have been developed in reaction to the seemingly involvement of street networks in patterning crime. Theoretical focus of methods has also been different: As one such example, they consider the concept of the social tension in their model, and Mohler et al. (2011) apply an analogy of the earthquakes to their statistical method of explaining crimes. The latter of them is the basis of a functioning predictive policing system which proves the practical usefulness of such modeling. Even several other phenomena have been considered although the majority of models which have been published are based on high-volume urban crime. Recent high profile rioting has seen an explosion of interest in the topic, and models have been proposed to deal with the cases in London and Paris. These models attempt to use behavioral concepts (selection of the target, the contagious behavior) and the effects of demographic parameters that change with space [17]. The target selection principles have also been applied to the field of maritime piracy to guide the defense strategies.

The Contacts and Gang Territory

The modeling problem of crime that is created between a pair of criminal groups is a fascinating problem; such as gang fighting and rivalry. This being a spatial matter with the significance of territory it is, there are strong parallels with spatial ecology; gangs resemble species in that they are resource and territory based competitors. The efforts of law enforcement to break gangs can be viewed as predation in this predator-prey relationship. To predict the changes of territory, manifestation of violence, and how to minimize the violent actions, it may be reasonable to learn about the dynamics of these populations. The model provides the optimum state of social control and the behavior it displays is consistent with the real-life statistics, like the abrupt increase in the gang numbers. More modern approaches have followed the same line, only they have concentrated on the hostile relationships within the gangs. Although certain people have considered the dynamics of gangs in an ecological perspective, it is not the only one. As one such example, we can point out the study by Barbaro et al. (2013), which examines the mechanism of territorial marking with the help of graffiti and establishes the circumstances in which individual divisions are formed through approaches to statistical physics. Such research may prove handy in the actual world as there are many violent outbursts that tend to erupt in such intersections. It has also been formulated in the point process formulation, particularly as a framework of retribution, on a parallel matter. The method of predicting missing data has proven to be promising particularly in the context of identifying unknown offenders of violent crimes.

Criminal Organizations

Due to the complex nature of most criminal activities, networks provide an effective conceptualization of these phenomena to examine and understand them. The most frequent form of contact that will be discussed here is collaboration, in the case of inorganised crime or terrorism, but has also involved antagonistic relationships such as gang rivalries. Within such contexts, conversations or data on co-offenders can be used to build and analyze networks using various sources of data. Various questions can then be answered with the aid of social network analysis including the extent to which a system is hierarchical, role identification and the assessment of influence. This has been done to several topics; some of these topics are trafficking, organized crime and terrorism. Organized criminal networks are very secretive and this makes them even harder to study. The characteristics of these businesses are to hide their relationship. Scholars in this area have suggested numerous ways of making probabilistic guesses regarding network topology in covert situations, and seek to locate a solution to this issue. One such field has been in terrorism where Dynamic Network Analysis has been applied with the aim of incorporating data provided by multiple sources through predictive network analysis techniques. One of the most urgent reasons to research the criminal networks is the hope to find the most effective way to interrupt them. Numerous studies have been conducted on the resistance of networks to attacks, and a wide variety of loose strategies, most of which involve elimination of major participants, such as arresting them.

Checking and Perseverance

This is a negative aspect of criminal networks, yet a welcome feature of security in many other areas. An example would be crime science strategy which would seek to enhance the ability to resist attack against potential targets especially those that are huge interdependent systems. This can be done with mathematical analysis of how such systems respond to various types of assault, as many such systems (such infrastructure) are most readily described as networks. This can be used to understand critical transitions, or where assaults are sufficient to result in catastrophic failure, and be able to locate weak spots. Network resilience to attacks has been studied longer, and in particular energy networks and key phenomena. The interdependence of internet and energy distribution networks, such as, has been a more recent study due to its supposed contribution to previous power blackouts. These types of interdependencies must be taken into account when conducting the analysis of the threats to such facilities due to the frightening prospect of disastrous collapses in such systems. Other than the research by Carvalho et al. (2014) that investigated the dynamics of energy networks to real security threats such as wars, there is practically no research that has looked specifically on the security implications of such a weakness. Vulnerability and protection has been well researched when it comes to the allocation of security resources to places that are perceived to be vulnerable. The two parties of the game that has been structured as an adversarial game are the terrorists and security agents. These types of games allow the aggressors to select between a set of targets, and those targets have their own advantages and disadvantages, and the role of the defender is to distribute resources to minimize the risk of each target. The fact that both parties can take into consideration the actions of the other party (say, terrorists can view the level of defense of each target) makes the conceptualization as a game a work. By solving these models it is possible to obtain the optimum allocation of defensive assets which, in the majority of cases, is a mix of geographical assignments. Two actual instances of the places where such tactics have been applied in inspection are the airports and transportation systems.

Financial Frameworks

The economic modeling of crime usually centers in the exploration of the degree of criminality within the population. In the broader sense of detecting malfeasance, one of the themes that have been researched in this area is inspection. With so much work required, it would be irresponsible not to discuss this, although it will not be as well aligned to the criminal science approach as the other topics addressed. Nevertheless, the assessment presented in the essay by Gordon (2010) is more detailed. Manning also explains this point in more detail in a separate chapter of this guide, but the work of Becker (1968) that initially initiated the field is an excellent demonstration of the general method. This theory holds that the behavior of criminals is rational since a social loss function, which quantifies the financial impacts of crime, is used to make the rational decision. In this case, the probation, severity and cost of punishment are taken into account in the number of crimes and damage they cause. One can create a state that is economically optimal by reducing this role which poses the question of how much crime should society put up with and how much resources to use in investigation and punishment. This is as an example of the macro-level perspective of these models; that is, the societal consequences are considered. Among the consequences, which have been researched in this way, are inequality, the effect of punishment, and the inclination to commit crimes. Glaeser et al. (1996) hypothesized that variations in the rates of crime by the US cities could be attributed to the influence of social contacts on crime, which was subsequently explained by other models. Studies involving the effect of punishment on the criminal organizations have also been conducted through an economic perspective. It can only be compared to epidemiology and more so the notion that criminality is a contagious disease bearing in mind the significance of interaction. Compartmentalization, where the population is stratified into multiple states (as in, e.g., the state of being susceptible and the state of being a criminal) and individuals transition among them is used by many people in the field including those who have developed models of criminal behavior. The arrival of compartmental models through the prism of game theory is also a relatively new trend. According to the example, there are four types of people, namely, informants, villains, paladins, and apathetics with different tendencies to criminal activities and possible evidence in a court of law. At the conclusion of every go-round of the game, each player is given rewards depending on their interactions (possible crimes). The moves of more successful players can be copied by simply moving between states after each round, and the objective of the model is to record the score of the number of players in each state. Although the significance of informants to a crime-free society should be regarded as the greatest lesson of the study, the flexibility of the model provides the opportunity to researchers to test a vast variety of methods and interventions by merely altering the incentive system. The economic models are helpful as they enable us to look at how the over-all effect of the change of policy influences crime on a macro level.

Picture analysis Applied Mathematics

Picture analysis Mathematical image processing has wide applications in many fields such as astronomy, medical imaging, video transmission and surveillance. Signals express images which are one-dimensional. Planar images are two dimensional and volumetric images are three dimensional. The distinction between the grayscale and color images is that the former is regarded as a vector valued function whereas the latter is regarded as a single valued function. The defects such as flaws such as noise and blurring reduce the quality of images.

Planar image is mathematically represented as a function of the spatial domain whose value is known.

$$(x, y) \to f(x, y)$$

The intensity is the quantitative way of measuring energy that is emitted in a picture by a physical source.

$$0 < f(x, y) < \infty$$

$$f(x, y) = i(x, y)r(x, y)$$

The pictures produced as we move through some medium can be characterized by a similar word.

$$rk = p(x, y)$$

$$p, k = 0,1,2, \dots L-1.$$

Le$s_k = m(x, y)$ be the desired gray level of output image m.
A transformation T :

$[0, L-1] \to [0, L-1]$ such that $s_k = T(r_k)$, for all $k = 0,1,2, \dots, L-1$.

$$m(x, y) = s_k = (L-1)\sum_{j=0}^{k} p(r_j)$$

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$\frac{\partial s}{\partial r} = (L-1)p_r(r) = T'(r)$$

$$\frac{\partial r}{\partial s} = \frac{\partial}{\partial s}\left(T^{-1}(s)\right) = \frac{1}{T'(s)} = \frac{1}{(L-1)p_r(r(s))}$$

$$p_s(s) = p_r(r)\frac{\partial r}{\partial s} = p_r(r)\frac{1}{(L-1)p_r(r(s))} = \frac{1}{(L-1)}$$

In the discrete model, the uniform probability distribution function ps of the interval 0, L1 is the same as a flat histogram.

Enhancement of Images

The second order partial derivatives of f whose constant Laplacian of an image are evaluated are defined as,

$$\frac{\partial^2 f}{\partial y^2}(x, y) \approx \frac{f(x, y+k) - 2f(x, y) + f(x, y-k)}{k^2}$$

$$\Delta f(x, y) \approx f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

Order filters are utilized in image processing. Order statistic filters are a non-linear spatial filter, which uses the ranking of the pixels within the image area it represents to choose the value to replace that of the pixel in the centre. Since it uses a linear combination of order statistics, it can estimate means. Now compared to the mean filter, what is its difference? A sliding-window spatial filter that uses order statistic in place of the mean value uses N observations, but ordered in ascending order, unlike the mean filter, which is a simple sliding-window spatial filter, whereby the center of the window is replaced by the mean of all pixel values.

$$X_1 < X_2 < \dots < X_N$$

$$a_i = \begin{cases} 1 & i = \frac{N+1}{2}, \\ 0 & otherwise \end{cases}$$

$$a_i = \begin{cases} \frac{1}{M} & \frac{N-M+1}{2} \le i < \frac{N+M+1}{2}, \\ 0 & otherwise \end{cases}$$

The Use of Convolution in IR

The filter effect of photos is brought about by convolution. A mathematical process involving integer operation is applied to a picture by application of a matrix operation. The first method of doing so is to sum up the weighted values of all the pixels surrounding a central pixel to determine its value. The end-product is a newly edited and filtered photograph. Convolution is applied in order to obtain a smoother, brighter, more intense or sharper image. In order to carry out a convolution, a color value of an individual pixel is multiplied with a matrix known as a kernel in reference to its neighbours. In convolution, the output is different in accordance with the size of the kernel and the number pattern within the kernel. Although the most popular size is 3x3, the kernel may be any size. The convolution formula is:

$$W = \frac{\sum_{i=1}^{q}\sum_{j=1}^{q} f_{ij} d_{ij}}{F}$$

W is the output pixel value.
f_{ij} the coefficient of the convolution kernel at position i,j in the kernel matrix
d_{ij} data value of the pixel that corresponds to f_{ij}
F sum of the coefficients of kernel matrix
q dimension of the kernel.

Forensic Picture Analysis has Mathematical Basis

Reliability of the source of information may be compromised in a situation, this may be where forensic analysis may provide the confidence. The forgery detection algorithms help identify the interactions between the real and altered features. The probability and linear algebra help in learning forgeries. Traumatic brain damage is analyzed with the help of a Fourier transform infrared micro spectroscope. Hypothesis testing based on local estimates is helpful to tell the difference between real and manipulated photos. Stochastic gradient descent is also used in the stochastic gradient descent techniques of forensic editing detection. The reason behind their use is that multimedia forensic is considered to be more efficient in cases when metadata cannot be relied upon [18]. Graph theory is also applied in some of the most modern studies to provide better picture recognition. Therefore, to understand the fundamental principles used in the field of forensic sciences, we will now attempt to describe the different components.

Steganography

It is referred as steganography, the art of secret writing. Compare and contrast this with cryptography, which is used as a method of keeping a message unintelligible even to the sender. This is referred to as steganography. One such steganographic capability has commercial application in the digital world and is known as digital watermarking. The artist will be the only person who can prove the ownership since only the artist can erase the watermark, which will give the artist the only chance to claim ownership in case another user of the files claims the file as his or her own. A computer forensics investigator is allowed to feel there is steganography based on meeting some criteria including the nature of the hardware or software present, what the suspect or witnesses in the library holds, huge amounts of apparently duplicated images, what the suspect or witnesses claims, among other pertinent considerations (Kessler cited). A site could be suspicious because of the demographics of the people it is serving or the content that it contains. Besides, these things could reveal passwords to the examiner [19, 20]. Also, steganography searching is important in areas beyond the intelligence gathering and criminal investigations. Statistical methods are very often used in the analysis of digital images which is determined by the probability of XN 1 n¼0 xn cos X 4Fx 2pi -1

N (30) (31) Forensic Analysis of digital images often relies on the use of statistics. One of them is a likelihood ratio based on scores (SLR) that may be utilized to detect camera devices. The score-based likelihood can be used to reduce evidence, including a pair of fingerprints found at the crime scene and the ones belonging to the suspect. The similarity score = 1- normalized correlation, photo-response non-uniformity resembles a fingerprint upon a camera [21, 22]. The process includes forgery and tampering detection issues, source identification issues. The single-lens reflex is relevant to the problem of identification of camera devices. It analyses the strength and weakness of the evidence that extends the conclusion and hence concludes whether a digital image was captured by a known camera device or not, in the case of an unknown image. It is applicable in diminishing the importance of the evidence. SLRs were used to fit both sets of scores using probability density functions. In order to calculate the SLR we will compare the two pdfs with the obtained score that is the difference between the noisy image of the camera fingerprint and the image. Suppose that there exists a noisy image (I1) and a noise-free image (I0). K represents the fingerprint of the camera, and ϕ represents all the other noise factors of the image. We can therefore model the output of the image t I1 as:

$$I_1 = I_0 + I_0 K + \phi$$

$$W^{(i)} = I_1^{(i)} - F\left(I_1^{(i)}\right)$$

$$\hat{K} = \frac{\sum_{i=0}^{N} W^{(i)} I_1{}^{(i)}}{\sum_{i=0}^{N} \left(I_1{}^{(i)}\right)^2}$$

In order to make sure that the similarity score is correct, the computation of the normalized correlation is replaced with the peak to correlation score. The former have better stability of the decision threshold. The other alternative that can be adopted is a likelihood ratio based on a trace anchored score. The SLR framework uses the hypothesis testing method in order to support its findings with facts.

Introduction to neural networks: The theory of neural networks

It is based on the architecture of the human brain to interpret information and adjust to it. Its approximation property is far-reaching. Some of the numerous types of neural networks include recurrent, convolutional, radial basis function, feedforward, and modular neural networks. We shall begin by briefly reviewing its basic structure so as to understand its architecture better. Based on the input xj, the weights α1j which are functions of j between 1-3 are utilized to generate the output y1.

$$y_1 = f(\alpha_{11}x_1 + \alpha_{12}x_2 + \alpha_{13}x_3)$$

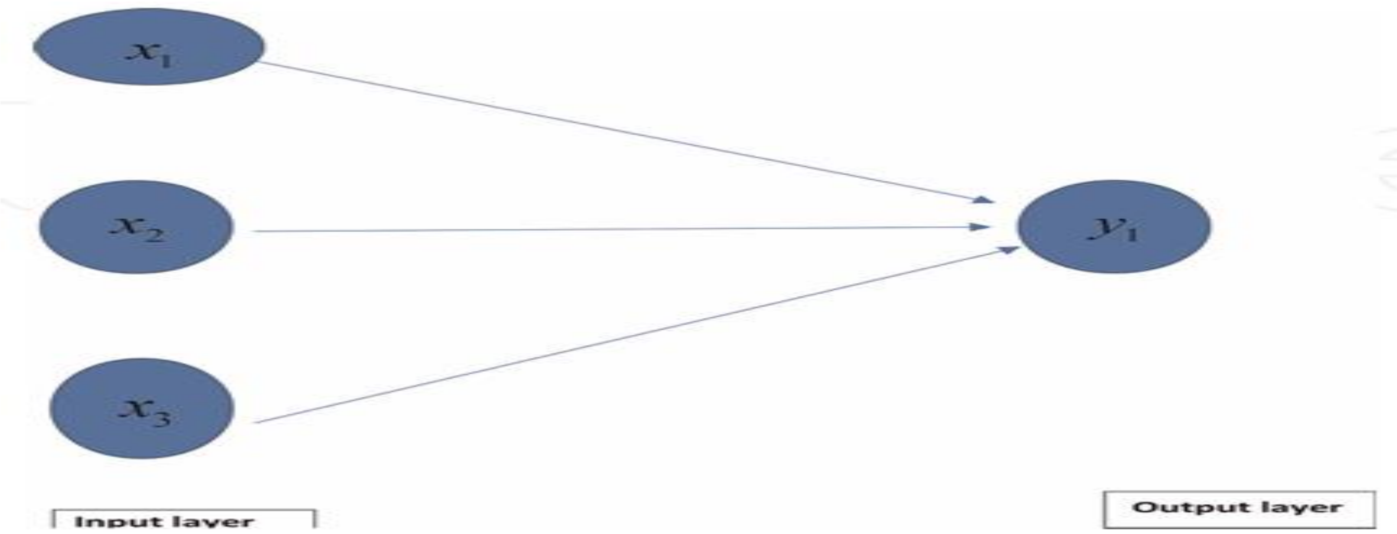

Figure 1. Representation of a single layer neural network not having a hidden layer.

Such kinds of structure are classified as single-layer feedforward networks. It can enhance its computing capabilities by introducing a hidden layer or a number of layers on a multilayer feed forward network. To complete the loop and get an enhancement in the process, one of the inputs in the feedback networks is the output. Moreover, there are other patterns that can be used to enhance the performance of the network. According to Kingston who referred to simulation experiments, the pattern of tool marks (including the pattern of bullet striation) is stored in a Hopfield net, a type of neural network which, under input of a different mark with the same tool, recalls the corresponding pattern in the same tool [23-26]. The convolutional neural network is one variety of deep learning method that is capable of differentiating between features, emphasizing different regions of an image, and taking the image as an input. Its form is highly comparable to brain neurons. An input picture into a multilayer perceptron is fed into the picture through a transformation to a picture into a 9*1 vector when it is being classified. To analyze it more accurately, digital forensics employs the neural networks [27-31].

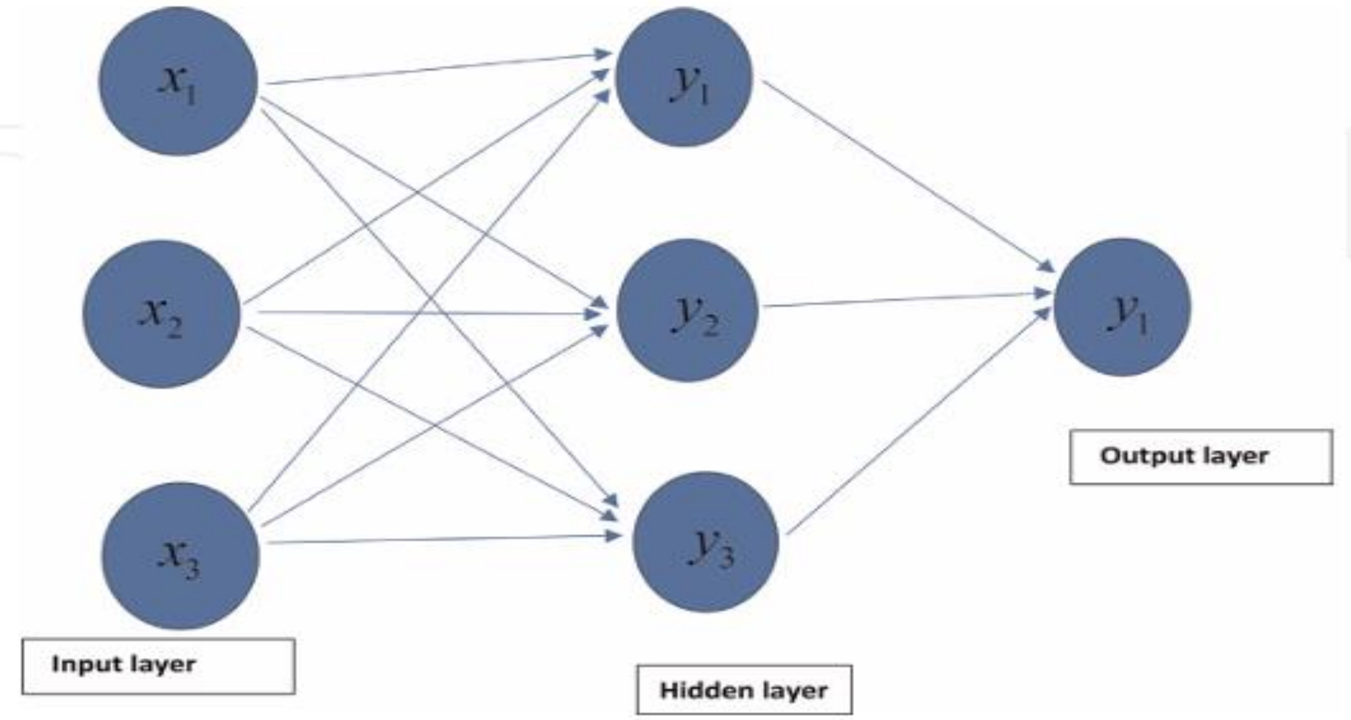

Figure 2. The neural network with one hidden layer that has three inputs and one output with one hidden layer can be visualized.

Probability fundamentals

Forensic analysis is based on probabilistic reasoning and inferences. Statistics is preoccupied with the collection and description of numbers, and probability is rather a branch of logical research. It has the potential to measure. Forensic science can be applied in criminal cases where chemical analysis of the suspect materials can be done and the elemental composition of the broken glass can be measured. The mathematical subfield of probability (Aitken-Roberts-Jackson) [3235] aims to have a framework of thinking about and making choices under uncertainty. Criminal justice only has one possible outcome; the accused is either literally guilty or innocent. This again implies that $p(G) = 1-p(I)$; and $p(I) = 1-p(G)$ by the usual laws of mathematics. The probabilistic equations may also be used to measure uncertainty levels associated with certain estimates. Probability is extensively applied in AI because it is concerned with data collection, processing, and analysis questions that result in solid conclusions. With AI becoming mainstream in most fields, including farming, engineering, demography, healthcare, education, marketing, and others, probability-based algorithms are likely to acquire the growing significance. In the following section we shall briefly glance at some of the most fundamental concepts. The task of characterization and analysis of data will be the prerogative of statistics [3639]. It includes classification and testing hypothesis selection. Included here is a synopsis. The thing about an experiment is to measure and observe. In experiments under discussion, the most significant one is the inclusion of randomness. The subgroups of a statistical sample are events. P(A) stands out as the probability of event A to occur under the assumption that all other events have the same likelihood of occurrence and finite outcome:

$$P(A) = \frac{\text{Number of outcomes to the occurance of an event } A}{\text{Total number of equally likely events}}$$

$$0 \leq P(A) \leq 1$$

There must be a certainty even when P(A)=0 and P(A) =1 which is impossible. Since event B has already occurred, there is need to explore the possibility of event A in this setting as articulated by:

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Conclusion

The basis of AI is identical cognitive functions of people: learning and retrieval. A network is expected to recognize a pattern already learned even in the case of some noise. Associative memory is helpful in construction of multimedia databases. In this context, optimization that is aimed at finding a solution that satisfies a series of constraints is important. The final objective of artificial intelligence is to develop models that can be acceptable by the human brain. The idea is to construct computational structures which mimic neurons or systems of neurons and to connect these structures to each other to form neural networks. This chapter will attempt to expose the reader to the basics of artificial intelligence (AI) and how it can be applied in forensic science by use of mathematics. To implement the methods, the subjects might have to be analyzed in details. This will be a very important move towards understanding and exploring other search methods. Some of the search strategies applied to solve problems are blind search, searching in extent, and heuristic search. Providing the reader with an introductory knowledge of graph theory and probability and demonstrating its use by playing games such as chess and go is an incentive to learn more. It is possible to view research into Bayesian networks, e.g., in terms of a direct acyclic connected graph, where the nodes are associated with a probability distribution that characterizes the association between the node and the edges. The above strategies can be used to create data modeling, with the assistance of MATLAB. As an example, assuming that we are aware of the characteristics that have the strongest effect on the energy load, then we can use curve fitting or statistics to model the data through either linear or nonlinear regression. Machine learning algorithms such as a neural network or a decision tree can be used when the system under investigation is highly complex, when a large number of variables are involved, or when the equations are not known. Application of Filion neural fitting application. Further conceptualization will serve the purpose of bettering the current researches to the advantage of society. The amount of potential growth in the sphere of Internet of Things forensic investigation as of the future is immense, due to the fact that different algorithms can be precisely optimized through incorporating information on the relevant inputs, which may reflect the scope of the prototype. Ultimately, the requirements in the chapter will assist the readers to address the basic concepts in the depth, which will further research in the field of image processing and forensic sciences. It may be possible to improve AI in the future by investigating forensic image processing algorithms, but it can not until we have a clear understanding of the principles of the processes that we have discussed. The topics discussed here might attract the attention of non-forensic scientists because of their widespread use in the engineering field.

One of the main objectives of crime science is the use of the same formality and rigor with the study of crime that has traditionally been applied to the physical sciences. It is therefore no wonder that mathematics is so extensively found in crime science, as both fields are very dependent on mathematics in terms of language and equipment. This chapter has attempted to give an outline of how mathematics could play a specific role in the sphere of crime research by examining potential applications and describing the most probable mathematical methods to be used. It is rather apparent that the chapter in question is indicative of classical mathematics being by no means the only discipline that pertains to the process of criminal science. There is a relationship between mathematical sophistication and an increased interest in crime modeling; many of the discussed methods, including network science, are more recent innovations, well linked with the increase in computing power and availability of data. It is estimated that mathematical approaches to crime study will continue to expand in scope and depth due to future advances in the computational social science and allied disciplines. Researchers have addressed numerous criminal problems, including the size and structure of criminal organizations, space-time patterns of crime, among many others. It is not hard to imagine that quantitative modeling can be applied in forecasting policing, e.g. in this case, without mathematics providing a unique perspective. Without mathematics, formalization and testing of the underlying ideas would be much harder in most of the other examples provided but it would not in any way necessarily make them null and void. It would be fair to revert to the main theme of this chapter, the extent to which such approaches bring the goal of crime science, as such formalization is clearly not an end in itself. Certainly, there is room to be improved in the aspects of aligning the field with the objectives of crime science, although the examples presented below demonstrate the apparent potential it has. In fact, mathematical modelling would provide an ideal context in which the scientific method could be applied to criminological phenomena: as above noted, it is the language of nature of encoding, analysing, and even proving ideas false. Nevertheless, the extent to which this is a genuine reality remains unknown particularly when it comes to the comparison of models and real-world data. The success is easy to observe in the models of simulating typical criminological events (including hotspots development) is that the underlying behavioral theories, which these models represent, are right. However, it remains doubtful to what extent these approaches represent reality due to the fact that they are usually created in extremely idealized situations, e.g. two-dimensional ones. The problem of making a difference between the possible models will have possibly even greater significance. The matter of which of these approaches is correct naturally arises when various approaches have been proven to be effective to recreate a desirable behavior in various contexts; hot spot development is one of them. Since the desired action is as wide as areas of high risk of crime, it is difficult to come across a model that fails to produce these areas, thus the assessment is invalid. Efforts to emulate such generalised behaviours beg the question of whether the bar is being set too low and as such, it is too easy to justify the use of models. This is not only an issue when it comes to mathematical models, but it is also an issue of criminal science overall. As in the previous analogy, the criterion of such mathematical models can only be increased in case there is one. However, in most cases, the extent to which phenomena have been outlined in empirical research is not specific enough to identify the outcomes of various models. The methods of such fields can only be expected to aid in similar degrees when the principles such as hotspot are conceptualized with the same degree of criteria that is demanded when studying physical systems. As the sophistication of empirical procedures increases steadily, the constraints of stylized facts as modeling material will be broken in the foreseeable future. The practical usefulness of the mathematical methods is, of course, another sphere of interest on the field of the criminal science. This area still leaves much to be desired, though, should we ever succeed in reproducing interesting events in the lab, it will be only a question of whether we can apply this knowledge to achieve anything in the real world only after leaving academia. The results of predictive policing have been encouraging in the areas of crime prevention and it is a good example of how in real world activities models can be employed to guide activities. Whether or not such move is feasible remains a subject of contention within the context of most current policing regimes and such practical implementation remains under the infancy stage of research. The mathematical techniques discussed in this chapter require additional research on its possible practical application. Attempts to apply mathematics to the sphere of criminal science should focus more on a practical outcome in the nearest future. The approaches mentioned in this synopsis are encouraging and suitable to the topic, yet they can only help criminal justice when they are implemented in a real world. This might require a shift in focus toward less rigorous mathematical study, but it will ensure that mathematics does not become useless to criminal studies.

Recommendations

- **Should perform performance audits**: Mathematical and algorithmic methods of forensic investigations, particularly those that require artificial intelligence and machine-learning, must be carefully validated before they are applied in courtrooms. Auditing methods to ensure accuracy, bias, and repeatability is a nice concept that deserves independent validation before it is applied to large representative data to detect deep fakes, face recognition, or fingerprint matches. Regulatory frameworks might be applied to algorithmic tools in a similar fashion as it happens with biomedical device validation.
- **Elevate inter-disciplinary collaboration in model construction:** Digital reconstruction, genetic genealogy, postmortem interval estimation, and anthropology are just but a few examples of the disciplines that will often have to collaborate. To illustrate, to approximate the time of death, the models of the differential equations should be modified by the real data of the environment. The presentation of the statistical evidence in court must be standardized to minimize ambiguity and enhance the understanding of the jurors. This involves how professionals express doubt in their testimony. It is also advisable that the expert witness should adhere to some guidelines that may encourage the application of the visuals like probability charts, confidence intervals and likelihood rato scales to distribute their findings clearly. The same must be done with other areas of forensic science, including the use of digital forensics, bite marks, and trace evidence to enhance the dissemination of forensic evidence in the same way the likelihood ratio framework enhanced the dissemination of DNA evidence.
- **Author accessible, clear and easy to understand forensic algorithms:** This is a prerequisite of any process whose results depend on machine learning to assess evidence. When black-box algorithms are under cross-examination, it is difficult to defend them. This should be made possible by experts capable of drawing clear and rational conclusions using mathematical reasoning, and developers and forensic laboratories must focus on explainable AI (XAI) technologies to achieve this. The public open-source toolkits and publicly available validation datasets could be used to improve accountability and reproducibility. Judges and prosecutors among other legal practitioners do not always receive formal training on statistical thinking. In order to resolve this, institutionalization of forensic statistical training is to be developed. Consequently, courses on probability and statistics have to be taught in forensic science and law schools as a compulsory course. They should be provided with constant

professional training to ensure that the practitioners and the court officials are able to interpret probability findings, statistical error rates, and likelihood ratios appropriately. It is suggested that forensic statisticians should be also actively engaged in casework: Last but not least, it is also inevitable that the forensic statisticians should be integrated into the investigation and judicial processes on a permanent basis. Their knowledge is not only necessary to play key roles in deciphering complex pieces of evidence, but also assists with advisory roles on the design of forensic experiments, the evaluation of alternative hypotheses, as well as ensuring that the arguments presented statistically in court are valid and within the realms of science. This will result in the reduction of room to make mistakes and increased trust in the justice system.

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